Extending Freeze-LTL on Multi-Attributed Data Words with Quantifiers

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Contents

Background and Related Work

- *Freeze LTL*: LTL with *freeze* and *check* operator
- Allows to store values in *registers* and compare with other values
- Checked over *data words*
- [Demri et al., 2005]: Not decidable in general!
- More than one register or *past-time operators* yield undecidability
- [Demri and Lazic, 2009]: Forward fragment with one register ($\text{LTL}_1^{\downarrow}$) decidable

A data word:

$\{p,q\}$	$\{p\}$	$\{p\}$	$\emptyset$	$\{q\}$	$\dots$
9	5	4	11	6	$\dots$

Background and Related Work

[Figueira, 2012]:

- Investigation of *Alternating Register Automata* (ARA)
- Introduction of $\exists_{\geq}$ and $\forall_{\leq}$ operators
- New, simpler proof for decidability of emptiness of ARA
- Uses proof technique based on *well-structured transition systems* [Finkel and Schnoebelen, 2001]
- $\Rightarrow$ Shows decidability of $\mathrm{LTL}_1^{\downarrow}$ with $\exists_{\geq}$ and $\forall_{\leq}$ via ARA

[Decker and Thoma, 2015]:

- Define new logic $\mathrm{LTL}_A^{\downarrow}$ based on $\mathrm{LTL}_1^{\downarrow}$
- Allows to store multiple values in each position and remains decidable!
- Requirement: *tree-quasi-ordered* attribute set and restricted comparison

Contribution of the thesis

- Take $\mathrm{LTL}_A^{\downarrow}[\mathbf{X}, \mathbf{U}]$ from [Decker and Thoma, 2015]
- Add quantifiers $\exists_{\geq}$ and $\forall_{\leq}^{x}$ from [Figueira, 2012]
- Call resulting logic $\mathrm{LTL}_A^{\downarrow}[\mathbf{X}, \mathbf{U}, \exists_{\geq}, \forall_{\leq}^{x}]$
- Adapt alternative proofs by Decker and Thoma from $\mathrm{LTL}_A^{\downarrow}[\mathbf{X}, \mathbf{U}]$ to $\mathrm{LTL}_A^{\downarrow}[\mathbf{X}, \mathbf{U}, \exists_{\geq}, \forall_{\leq}^{x}]$ to show decidability

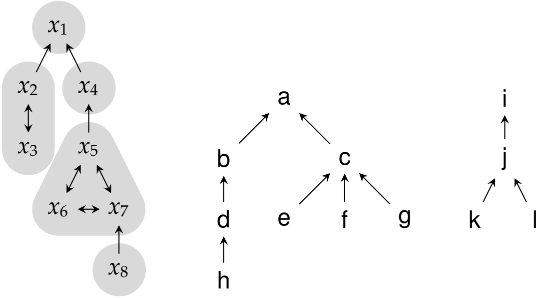
Tree-quasi-orderings

- $(A, \preccurlyeq)$ quasi-ordering + each downward-closure (*path*) total

  $\Rightarrow$ *Tree-quasi-ordering*

- All downward-closures linear $\Rightarrow$ *Tree ordering*

- min. elements = *roots*, max. elements = *leaves*



Data words

- $w = (a_1, \mathbf{d}_1)(a_2, \mathbf{d}_2) \cdots (a_n, \mathbf{d}_n) \in (\Sigma \times \Delta^A)^+$ is *A-attributed data word*
- consisting of tuples of *letters* $a_i \in \Sigma$ and *data valuations* $\mathbf{d}_i \in \Delta^A$
- map each attribute to some data value.
- If $A \approx [k]$, valuations may be called *vectors*

{a}	{b}	{a}	{c}	{b}
$x_1 \mapsto 5$	$x_1 \mapsto 5$	$x_1 \mapsto 8$	$x_1 \mapsto 8$	$x_1 \mapsto 5$
$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$	$\uparrow$
$x_2 \mapsto 3$	$x_2 \mapsto 3$	$x_2 \mapsto 3$	$x_2 \mapsto 2$	$x_2 \mapsto 2$

Syntax

Let A be fin. set of attributes, AP fin. set of atomic propositions.

Syntactically valid formulae in $\mathsf{LTL}_A^{\downarrow}[\mathbf{X}, \mathbf{U}, \exists_{\geq}, \forall^x_{\leq}]$ are described by

$$
\begin{array}{lclcccccccc}
\varphi & ::= & & p & \mid & \neg\psi & \mid & \varphi \wedge \varphi & \mid & \varphi \vee \varphi \\
 & & \mid & \mathbf{X}\varphi & \mid & \varphi \mathbf{U} \varphi & \mid & \downarrow^x \varphi & \mid & \uparrow^x \\
 & & \mid & & \exists_{\geq}\varphi & & \mid & & \forall^x_{\leq,\psi}\varphi & \\
\psi & ::= & & p & \mid & \neg\psi & \mid & \psi \wedge \psi & \mid & \psi \vee \psi \\
 & & \mid & \mathbf{X}\psi & \mid & \psi \mathbf{U} \psi & \mid & \downarrow^x \psi & \mid & \uparrow^x
\end{array}
$$

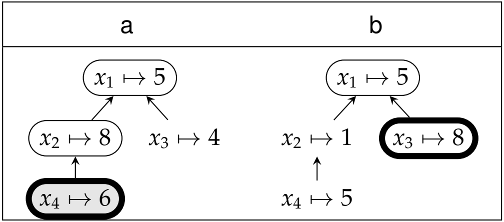
where $p \in AP$ and $x \in A$. Parentheses may be used freely.

More syntax

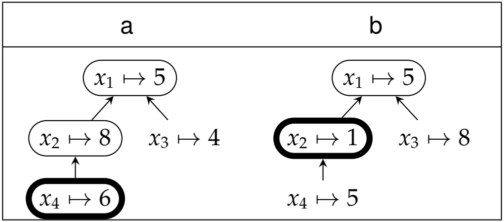
Some syntactic sugar for convenience:

$$
\begin{array}{rclrcl}
\text{true} & := & p \vee \neg p \quad p \in AP & \text{false} & := & \neg\, \text{true} \\
\varphi \Rightarrow \psi & := & \neg\varphi \vee \psi & \varphi \Leftrightarrow \psi & := & (\varphi \Rightarrow \psi) \wedge (\psi \Rightarrow \varphi) \\
\mathbf{F}\varphi & := & \text{true}\, \mathbf{U}\varphi & \mathbf{G}\varphi & := & \neg\mathbf{F}\neg\varphi \\
\overline{\mathbf{X}}\varphi & := & \neg\mathbf{X}\neg\varphi & \varphi\, \mathbf{R}\psi & := & \neg(\neg\varphi\mathbf{U}\neg\psi)
\end{array}
$$

Example of freeze and check



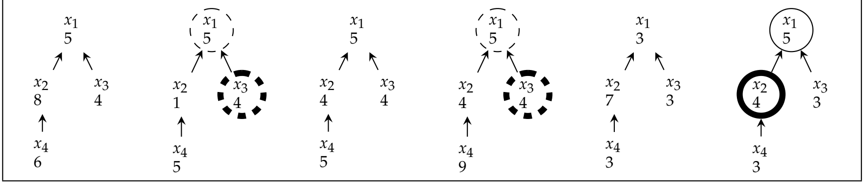
$\varphi_1 = a \wedge \downarrow^{x_4} \mathbf{X}(b \wedge \uparrow^{x_3})$



$\varphi_2 = \downarrow^{x_4} \mathbf{X} \uparrow^{x_2}$

Example of $\forall^{x}_{\leq,\psi}$ operator

1	*2*	*3*	*4*	*5*	*6*
c	a	c	a	c	b



$$\varphi = \mathbf{F}(b \wedge \forall^{x_3}_{\leq,a} \uparrow^{x_2})$$

Example of $\exists_{\geq}$ operator

b	a	b	a	a	b	a	a
1	1	3	1	3	2	2	1
↑	↑	↑	↑	↑	↑	↑	↑
2	2	2	2	2	3	3	3
↑	↑	↑	↑	↑	↑	↑	↑
3	3	1	3	1	1	1	2

$$\varphi = \exists_{\geq}((b \Rightarrow \neg \uparrow^3)\mathbf{U}(a \wedge \uparrow^3))$$

NRA execution example

$$\mathcal{A} = (\{a, b\}, 3, \{q_1, \dots, q_8\}, q_1, \delta)$$

q_i	1	2	3	4	5	6	7	8
$\delta(q_i)$	$q_2 \wedge q_3$	a	$\triangleright q_4$	$\mathsf{store}(q_5)$	$\triangleright q_6$	$q_7 \wedge q_8$	b	eq_2

$$w = \begin{array}{|ccc|} \hline \mathsf{a} & \mathsf{a} & \mathsf{b} \\ \hline 4 & 5 & 5 \\ \uparrow & \uparrow & \uparrow \\ 5 & 1 & 1 \\ \uparrow & \uparrow & \uparrow \\ 7 & 2 & 9 \\ \hline \end{array}$$

$$\Rightarrow \mathcal{C}_1 = (1, \triangleright, (a, (4, 5, 7)), \{((4, 5, 7), q_1)\})$$

$\mathcal{A}$ obviously corresponds to formula $\varphi = a \wedge \mathbf{X} \downarrow^2 \mathbf{X}(b \wedge \uparrow^2)$!

WQO and Transition Systems

Well-quasi-ordering: Let $(M, \preccurlyeq)$ be a quasi-ordering.

$(M, \preccurlyeq)$ is called *well-quasi-ordering*, if every infinite sequence of elements $m_1 m_2 m_3 \cdots$ from M contains two elements m_i, m_j, so that $m_i \preccurlyeq m_j$ and $i < j$.

Transition systems:

- $(S, \rightarrow)$ is *transition system* with set of states and trans. relation
- Succ(s) denotes the set of direct successors of state $s \in S$
- If Succ(s) is finite for all $s \in S \Rightarrow$ *finitely branching*.
- If Succ(s) is computable for all $s \in S \Rightarrow$ *effective*.

Reflexive Downward Compatibility

A transition system with a wqo relation $\leq \subset S \times S$ is called *reflexive downward compatible* wrt. $\leq$, if and only if for all $a_1, a_2, a_1' \in S$ with $a_1 \to a_2$ and $a_1' \leq a_1$ there exists a_2' with $a_2' \leq a_2$ and either $a_1' \to a_2'$ or $a_1' = a_2'$.

$$\forall a_1, a_1', a_2 \quad \exists a_2' :$$

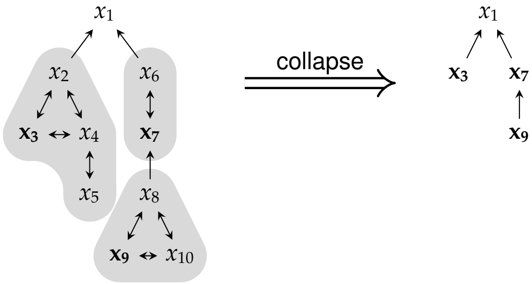
$$\begin{array}{ccccccc} a_1 & \to & a_2 & & a_1 & \to & a_2 \\ \text{|}\!\vee & & \text{|}\!\vee & \text{or} & \text{|}\!\vee & & \text{|}\!\vee \\ a_1' & \to & \mathbf{a_2'} & & a_1' & = & \mathbf{a_2'} \end{array}$$

Decidability of NRA emptiness – proof sketch

- Show that configurations of NRA give rise to a wqo:
  - Successively construct wqo relations on (parts) of configurations
  - Apply corresponding results where appropriate
- View transition relation of NRA configurations as *transition system*
  - Show that NRA transition relation is rdc wrt. the configuration wqo
  - $\Rightarrow$ effective, finitely branching *downward-WSTS with refl. compatibility*
- Apply result from [Finkel and Schnoebelen, 2001] showing that reachability of accepting configurations is decidable

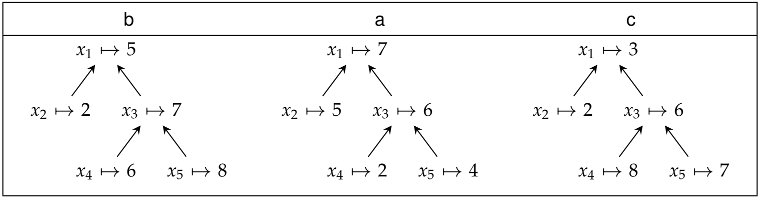
Collapsing of the SCCs

Let A be a tree-quasi-ordered set of attributes. Every LTL$^{\downarrow}_{A}$ formula φ can be translated to an equisatisfiable LTL$^{\downarrow}_{A'}$ formula φ', where A' is a tree ordering.



Frame encoding

Let A be a tree ordering, $k = \text{ht}(A)$ be the depth of A and $w \in (\Sigma \times \Delta^A)^+$ a data word of length m. We can translate the A-attributed data word w to a $[k]$-attributed data word w'.



encode ⇒

b	b	b	a	a	a	c	c	c
5	5	5	7	7	7	3	3	3
↑	↑	↑	↑	↑	↑	↑	↑	↑
2	7	7	5	6	6	2	6	6
↑	↑	↑	↑	↑	↑	↑	↑	↑
1	6	8	9	2	4	5	8	7

Linearisation - sketch

If A is a tree-quasi-ordered set of attributes of depth k, then every $\mathrm{LTL}^{\downarrow}_{A}$ formula can be translated into an equisatisfiable $\mathrm{LTL}^{\downarrow}_{[k]}$ formula.

- Let Φ be an $\mathrm{LTL}^{\downarrow}_{A}$ formula
- Applying collapsing assume formula over tree-ordered attribute set
- Argue that each model over A attributes has a model over $[k]$ attributes (frame encoding)
- Assume formula in a normal form with $\downarrow$ only before $\uparrow$ or $\mathbf{X}$ (applying some simple equivalences to "push" freeze operators to the right)
- Extend formula with additional terms to ensure that the frame encoding is correct

Linearisation - sketch

Translate formula recursively to get resulting formula $\hat{\Phi} = t(\Phi) \wedge \beta_1 \wedge \beta_2 \wedge \beta_2$:

$$
\begin{aligned}
t(a) &:= a \\
t(\neg\psi) &:= \neg t(\psi) \\
t(\psi \wedge \xi) &:= t(\psi) \wedge t(\xi) \\
t(\mathbf{X}\psi) &:= \wedge_{j=1}^{n} p_j \Rightarrow \mathbf{X}^{n-j+1} t(\psi) \\
t(\psi \mathbf{U} \xi) &:= (p_1 \Rightarrow t(\psi)) \mathbf{U} (p_1 \wedge t(\xi)) \\
t(\downarrow^x \psi) &:= \mathbf{X}^{\mathsf{sb}(x)-1} \downarrow^{\mathsf{ht}(x)} t(\psi) \\
t(\uparrow^x) &:= \wedge_{j=1}^{n} p_j \Rightarrow \mathbf{X}^{\mathsf{sb}(x)-j} \uparrow^{\mathsf{ht}(x)} \\
t(\exists_{\geq}\psi) &:= \exists_{\geq} t(\psi) \\
t(\forall^x_{\leq,\psi}\xi) &:= \forall^{\mathsf{ht}(x)}_{\leq,\psi \wedge p_{\mathsf{sb}(x)}} t(\xi)
\end{aligned}
$$

Translation to NRA - sketch

Every $\mathrm{LTL}^{\downarrow}_{[k]}[\mathbf{X}, \mathbf{U}, \exists_{\geq}, \forall^{x}_{\leq}]$ formula φ can be translated into a corresponding k-NRA $\mathcal{A}_{\varphi}$, so that for every non-empty data word $w \in (\Sigma \times \Delta^{[k]})^{+}$:

$$w \text{ satisfies } \varphi \Leftrightarrow \mathcal{A}_{\varphi} \text{ accepts } w$$

- take care of $\forall^{x}_{\leq,\psi}$: $\varphi' := \bigwedge_j \eta_j \wedge \varphi, \quad \eta_j := \mathbf{G}(\psi_j \Rightarrow \downarrow^{k} \mathbf{G} \text{ true })$
- $\Rightarrow$ carry along all data values pre-filtered by different ψ_i in formula
- convert ϕ' to negative normal form (neg. only before propostions and $\uparrow^{x}$)
- define a state for each subformula
- $\Rightarrow$ each state represents outermost token of rest-formula to verify
- define corresponding transition function

Decidability of $\mathsf{LTL}^{\downarrow}_{A}[\mathbf{X}, \mathbf{U}, \exists_{\geq}, \forall^{x}_{\leq}]$

$\mathsf{LTL}^{\downarrow}_{A}[\mathbf{X}, \mathbf{U}, \exists_{\geq}, \forall^{x}_{\leq}]$ is decidable if and only if A is a tree-quasi-ordering.

- ($\Rightarrow$) follows from [Decker and Thoma, 2015, Theorem 2]
  ($\mathsf{LTL}^{\downarrow}_{A}[\mathbf{X}, \mathbf{U}, \exists_{\geq}, \forall^{x}_{\leq}]$ is superset!)
- ($\Leftarrow$) follows from
  - linearisation
  - translation
  - decidability of NRA emptiness

Conclusion

Example formula using ordered attributes (**pid** “depends” on **res**):

$$\mathbf{G}(\mathsf{lock} \Rightarrow \downarrow^{\mathsf{pid}} ((\mathsf{use} \land \uparrow^{\mathsf{res}} \Rightarrow \uparrow^{\mathsf{pid}}) \land \neg\mathsf{halt})\mathbf{U}(\mathsf{unlock} \land \uparrow^{\mathsf{pid}}))$$

... and now we can also express properties like:

$$\exists_{\geq}\mathbf{F}((\mathsf{lock} \land \uparrow^{\mathsf{pid}}) \land \neg(\mathsf{use} \land \uparrow^{\mathsf{res}})\mathbf{U}(\mathsf{unlock} \land \uparrow^{\mathsf{pid}}))$$

“at some point a resource is locked, but not used”

$$\mathbf{G}(\mathsf{lock} \Rightarrow \forall^{\mathsf{pid}}_{\leq,\mathsf{lock}}(\uparrow^{\mathsf{res}} \Rightarrow \uparrow^{\mathsf{pid}}))$$

“each resource is always locked by the same process”

Open questions

- Lifting the logic to data trees
  - Figuiera has also results for Alternating Tree Register Automata (ATRA)
  - Add nesting to ATRA in same way?
  - Maybe interesting in context of XPath query validation
- Extending the logic with a linear ordering over the data values
  - Figuiera proved that linear ordering on data domain is decidable
  - $\Rightarrow$ Extend logic with operators like $\uparrow^x_<, \uparrow^x_>$?
  - Would allow better value inspection, e.g. $\mathbf{G}(\downarrow^{\mathsf{var}} \mathbf{X} \uparrow^{\mathsf{var}}_>)$.
- Analyzing the complexity of the logic
  - $\mathsf{LTL}^{\downarrow}_A[\mathbf{X}, \mathbf{U}]$ is not primitive-recursive
  - $\mathbf{F}_{\varepsilon_0}$-complete in *fast-growing complexity classes* [Schmitz, 2013]
  - Do $\exists_\geq, \forall^x_{\leq,\psi}$ affect this?

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